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An Open-Ended Approach in Developing Students' Proficiencies in Geometry

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Abstract

Aim: This study examined the use of open-ended approach in developing students' proficiencies in Geometry.
Methodology: The research design was a descriptive-experimental design with a single group pre-post research design, aiming to establish the use open-ended ended approach in developing students' proficiencies in Geometry. The study focused on developing mathematical proficiencies through the use of open-ended problems as engagement, assimilation, and enrichment activities. The respondents consisted of 38 Grade 9 students from a heterogeneous section in San Isidro National High School, selected through cluster sampling. The research instrument used was a pretest and posttest, which were carefully validated in terms of content and structure. The research procedure involved the preparation and validation of instruments, obtaining permission from the school, administering the tests, implementing the open-ended approach, conducting posttest and checking of students' answers using the modified CCSS Standard for Mathematical Practice Rubric. The collected data were subjected to statistical treatment, including calculating the mean and percentage for descriptive data and using paired t-test for inferential analysis.
Results: The teacher is generally effective in implementing an open-ended approach in the classroom. However, there is a need for improvement, particularly in promoting rich mathematical discussions, analyzing different strategies, challenging students to more difficult problems, and writing students' names beside their solutions.The pretest and posttest scores performance in Geometry of students who were exposed to an open-ended approach have developed their proficiencies. There was a significant improvement in student's performance in all of the proficiencies. Futhermore, There were highly significant improvements in students' mathematical proficiency in various mathematical practices after using the open-ended approach.
Conclusion: Findings revealed that utilizing an open-ended approach in teaching Geometry is an effective strategy for enhancing students' mathematical proficiency. The pretest and posttest scores revealed a significant improvement in students' performance across all proficiencies.

Keywords: : open-ended approach, proficiencies, standard for mathematical practices

INTRODUCTION

Mathematics is a subject that has an impact on life at any age and in any situation. Problem-solving is a critical component of mathematics and problem-solving skills is essential for students' success in mathematics and in their everyday lives. The National Council of Teachers of Mathematics (NCTM) continuously emphasizes the importance of problem-solving, and international trends in mathematics teaching have shown an increased focus on problem-solving and mathematical modeling. Problem-solving is being emphasized in standard for mathematical practices across grade levels. The NCTM and Common Core State Standards (CCSS) have incorporated process standards that emphasize problem-solving, reasoning, communication, connection and representation skills. These process standards connect other knowledge and skills, enabling students to be successful problem solvers and use mathematics in everyday life.

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The objective of this study is to determine the impact of open-ended approach. An open-ended approach in mathematics allows students to think actively and creatively when solving problems. It focuses on the process of problem-solving rather than just the product, fostering a growth mindset and diversity of thought.

The Theoretical Framework of this research is based on a constructivist perspective of mathematical understanding. Hiebert and Carpenter (1992) introduced a framework that sheds light on analyzing various aspects related to comprehending mathematics from a constructivist standpoint. They distinguish between external and internal representations of mathematical concepts, emphasizing that people need to represent these ideas in some form to think about and communicate them effectively. External representations, like spoken language, written symbols, drawings, or physical objects, play a crucial role in facilitating communication. Through expressing their mathematical ideas, individuals can begin to grasp the intricacies and appreciate the precision that sets mathematics apart from natural language. By mastering the precise language of mathematics, students can better express their thoughts and reveal nuances that may be concealed in natural language.

Regarding problem-solving, scholars define it as a higher-order cognitive process that involves significant intellectual abilities. Therefore, for this study, an approach centered around mental processes is more appropriate in addressing the research questions (Resnick & Glaser, 1976; Sternberg, 1982).

This study draws its foundations from Newell and Simon's (1972) information-processing (IP) learning theory, which highlights the similarities between human problem-solving and artificial intelligence. The theory emphasizes the importance of working memory capacity and the cognitive retrieval of relevant information. According to Newell and Simon, successful problem-solving relies on several factors related to the human information processing (IP) system, and this higher-order learning theory has been employed to develop cognitive processes for problem-solving.

The open-ended approach to learning, which was developed in Japan around the 1970s by Shigeru Shimada, Toshio Sawada, Yoshiko Yashimoto, and Kenichi Shibuya, plays a significant role in this research (Muhsinin, 2013). This approach, also known as an incomplete problem, presents problems with multiple correct answers. It allows students to gain experience in discovering, recognizing, and solving problems using various techniques (Shimada, 1997). Through the open-ended approach, students encounter problems with multiple solutions, fostering creativity and enabling the discovery of new solutions by combining their existing knowledge and skills (Shimada & Becker, 1997).

The open-ended approach to learning involves the use of open-ended problems, which differ from routine problems. These open problems fall into three categories: problems with open processes (multiple answers), problems with open products (multiple correct solutions), and problems with open development paths (solutions can be modified by altering conditions from previous solutions) (Sawada, 1997).

In 2010, the Common Core State Standards for Mathematics (CCSSM) were introduced as a comprehensive set of educational standards, widely adopted to create a unified foundation for mathematics education. The primary objective behind developing these standards was to ensure that all students, regardless of their location, possess the necessary skills and knowledge to excel academically in mathematics. The Common Core State Standards Initiative includes eight principles of mathematical practice, known as Standards for Mathematical Practice, which aim to cultivate various abilities that mathematics educators should foster in their students at all educational levels. These practices are based on well-established approaches, essential processes, and proficiencies deemed significant in the field of mathematics education (Gewertz, 2012).

The Standards for Mathematical Practice were formulated by drawing inspiration from two key sets of criteria. The first set comprises the NCTM process standards, which include problem-solving, reasoning and proof, communication, representation, and making connections. The second set of criteria stems from the strands of mathematical proficiency outlined in the National Research Council's report titled "Adding It Up." These strands encompass adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (McCallum, 2011).

In essence, the CCSSM and its associated Standards for Mathematical Practice seek to establish a common and robust foundation for mathematics education, ensuring that students receive a comprehensive and well-rounded mathematical learning experience, preparing them to tackle academic challenges effectively.

Mathematics is an established discipline in every curriculum worldwide. However, there has been a decline in student performance in mathematics for over 20 years, according to the Trend International Mathematical Science Study Advanced (TIMMS). No improvements have been discovered in the countries studied (Maltese & Tai, 2011). Thus, global action is needed to address this issue.

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The Common Core established standards for what content students should learn in mathematics based on their grade level. Moreover, it is crucial to develop students' habits of mind. The Standard for Mathematical Practice (SMP) was designed to provide educators with a common language for discussing mathematical thinking. However, teachers need help understanding and implementing these mathematical practices.

First and foremost, teachers need to take the time to examine the standard and determine what it truly means. It may be challenging to see these mathematical practices are related and how to distinguish one from another, what is new to these standards, and what it looks like in the classroom. (Education Development Center, Inc., 2016)

The standard focuses on developing students' core mathematical proficiencies through best practices in the classroom. There are eight standards in the mathematical practices' competencies. According to the literature, teachers frequently need help naming the standards for mathematical practice, misinterpreting standards, using them inconsistently in their classroom, and needing more proficiency in using standards as mathematics learners. (Edward, 2021)

Teachers play a crucial role in implementing the standards. According to surveys, most math teachers support and are willing to teach the standards. However, transitioning from previous to new standards may be challenging for teachers. (Reborn, 2013) Teachers play an essential role in establishing the standard for mathematical practices. They need to encourage them to implement and teach the students.

The Philippines' mathematics framework, developed by the Science Education Institute (SEI) of the Department of Science and Technology (DOST) and the Philippine Council of Mathematics of Teachers Educators (MATTED), aims to produce mathematically empowered citizens by developing students' critical and problem-solving skills. (SEI-DOST & MATHTED, 2011).

According to PISA results, Filipino students' mathematical literacy is below the target proficiency of the objectives of mathematics education in the Philippines. Similarly, the United States promotes the standard mathematical practice because their mathematical literacy result is lower than expected. Suitable learning methods can impact students' success in improving learning outcomes. (Irawan & Surya, 2017).

In the school year 2017-2018, mathematics had the lowest mean percentage score of 35.34, and problem-solving had the highest mean percentage score of 48.16 on the National Achievement Test (NAT) for grade 10 students in the Philippines.

However, students' problem-solving performance is still below the acceptable mean percentage score.

In San Isidro National High School, mathematics had the lowest mean percentage score, with 39.79 learners in the different subject areas in the first quarter of the school year 2022-2023.

Additionally, 38 out of 269 Grade 9 learners were non-numerates according to the pre-test result of the numeracy test. This evidence suggests that numeracy skills must be supported to help children become confident and capable learners as they navigate the increasingly complex global community.

Furthermore, implementing appropriate and effective learning strategies in the classroom will empower students' potential. It is necessary to plan and learn by the characteristics of mathematics and the characteristics of learners. During this time, open-ended mathematical problems and methods were rarely used in school-based mathematics education (Johnson, 2013). The open-ended approach gives students more opportunities to gain knowledge and experience and recognize and solve problems. As a result of this approach, problems are set with various methods and multiple solutions. Thus, students will be more active and creative in problem-solving.

Research Questions

The main objective of this study is to examine the effectiveness of open-ended approach in developing students' proficiencies in Geometry. To address this objective, several research questions need to be answered.

1. How do respondents perceived the open-ended approach in terms of:

    1.1 Stimulate
    1.2 Support
    1.3 Stretch
    1.4 Strengthen
    1.5 Successfully evaluate

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2. What is the pre-performance and post-performance scores of students' proficiencies in Geometry using open-ended approach in terms of;

   2.1 Make sense of problems and persevere in solving them
   2.2 Reason abstractly and quantitatively
   2.3 Construct viable arguments and critique the reasoning of others
   2.4 Model with mathematics
   2.5 Use appropriate tools strategically
   2.6 Attend to precision
   2.7 Look for and make use of structure
   2.8 Look for and express regularity in repeated reasoning

3. Is there significant difference between the pre-performance and post-performance scores of students' proficiencies in geometry?

Hypothesis

Given the stated research problem, the following hypotheses were tested on 0.05 level of significance:
Hypothesis 1: There is no significance difference between the pre-performance and post-performance scores of students' proficiencies in geometry?

METHODS

Research Design

This research used a descriptive-experimental design with a single group pre-and post-test research design. Marsden and Torgerson (2012) stated that pre-test and post-test research designs (pre-experimental) with a single group evaluate causal relationships between intervention and outcome.

Furthermore, this research focused in developing mathematical proficiencies in Geometry through the implementation open-ended approach. These open-ended problems were used in Weekly Learning Plan as engagement, assimilation, or enrichment activities. In the period needed in this research, the open-ended problems always inserted as the primary activity to know if the students developed their mathematical proficiencies in geometry. Their statistical differences were used after the pretest, and posttest scores were gathered.

Population and Sampling

The respondents for this research consist of 38 students from heterogeneous section from Grade 9-Rizal in San Isidro National High School, Catanauan, Quezon, during the school year 2022-2023. Grade 9 Mathematics students were selected as respondents with cluster sampling. Grade 9- Rizal was used as experimental group taught by an open-ended approach. The researcher employed cluster sampling technique to select respondents in this study. The researcher selects one section of Grade 9 students in mathematics, with 38 students. Cluster sampling is a probability sampling technique in which researchers divide the population into available sections. The one section is made up of remote area of barangays in Catanauan, Quezon. The researcher's cluster in data gathering and administering was considered. Cluster sampling is a method in which the researcher creates multiple groups of people from a population with homogeneous characteristics and an equal chance of being included in the sample.

Instrument

The researchers used a test as an instrument, applying a pretest and posttest. The pretest was given before an experimental research study or open-ended problems. The posttest was given after the treatment or after using open-ended approach with mathematical proficiencies. The pretest and posttest tests contained mathematical problem-solving scoring guides which promote a standard for mathematical practices which validated by the external validators and internal validators.

The pretest and posttest instruments were carefully validated in terms of content and even the structure of the questions by the external validators and if it is parallel to the Table of Specification (TOS) and learning competencies in Mathematics 9.

The pretest question consists of problem-solving competencies in Mathematics 9, which consists of five problems aligned to the mathematical proficiencies. The posttest question consists of modified pretest questions.

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In checking students' answer the CCSS Standards of Mathematical Practice Rubric was used. The teacher assessed task that map well with various indicators of the rubric. This rubric indicates how students were proficient in each mathematical practice based on the pretest and posttest questionnaire.

Adopted survey-questionnaire was used to determine the perception of the student-respondents of using open-ended approach in developing students' proficiencies in Geometry.

Data Collection

The researchers prepared a pretest and posttest of problem-solving in Mathematics 9 validated by the external validators. A Weekly Learning Plan (WLP) in the format of IDEA exemplars was used in the experimental research. After preparation of instruments, the researcher consulted her adviser, statistician, subject specialist, and technical editor for suggestions and comments to review the tools used.

After validation of the instruments, the researchers modified the test questions based on suggestions, modifications, and corrections from the adviser and the research experts. The researcher asked permission from the School Principal of San Isidro National High School to conduct the research and pilot testing on the students who underwent learning competencies. The researcher asked for help from a statistician to check the reliability of the pilot testing. Refinement of the test question was considered before administering the test to the respondents. The test question was distributed to the respondents and retrieved on the agreed-upon date.

According to Betty (2015), the Open-Ended approach involves a series of steps or procedures that guide the learning process. These steps encompass problem presentation, understanding, problem-solving, comparing and discussing, and concluding. In this study, the Open-Ended approach was implemented by the following sequence of the learning steps: Firstly, the teacher introduced the problem to the students. Next, the students actively engaged in investigating and exploring the issue at hand. The teacher played an important role in observing and noting down the responses provided by the students. Subsequently, a class discussion took place, where student responses were analyzed and deliberated upon. Finally, the students summarized their individual learnings, consolidating the knowledge gained throughout the process.

In the open-ended approach, the teacher presented the problem to the student without determining the solution. The teacher should take advantage of the variety of problem-solving approaches available to provide students with hands-on experience in discovering something new based on prior knowledge, skills, and ways of thinking. The implementation took 6 weeks in the Third Quarter Period in the school year 2022-2023.

After implementing the open-ended approach, a posttest was given to participants of the study to determine whether the students' proficiencies are being developed.

The researchers compiled all the instruments and gathered all the pretest and posttest scores. The researcher summarizes the scores and give the data matrix to statistical service center for treatment. When the statistical service center gives the results, tables was prepared by the researcher and analyzed to broaden the knowledge of the main focus of the study.

The researcher ensures the confidentiality of the respondents' results and information. The data results in the pretest and posttest within the researcher and thesis adviser.

Treatment of Data

In this study, experimental and inferential statistics were used. Mean was used to respond to descriptive data on the perception of respondents in open-ended approach.The percentage was used to respond the descriptive data on the pretest and post-test scores performance of student-respondents on the mathematical proficiencies in Geometry. Moreover, The paired t-test was used to analyze the data for inferential questions of the pretest and posttest within groups.

Ethical Considarations

The researchers demonstrated an unwavering commitment to ethical research standards by prioritizing the welfare and interests of all individuals and organizations involved in the study. Thorough attention was given to ensuring informed consent/assent from participants, explaining the purpose and potential risks of the research transparently. To protect participants' privacy, rigorous measures were implemented to maintain anonymity and confidentiality, removing personally identifiable information and safeguarding data access. The researchers upheld fairness by including diverse participants and distributing benefits and burdens equitably. By adhering rigorously to ethical guidelines, the study's foundation was strengthened, leading to valuable and trustworthy findings, while fostering an environment of trust, respect, and fairness throughout the research undertaking.

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RESULTS and DISCUSSION

PART I. RESPONDENTS' PERCEPTIONS ON OPEN-ENDED APPROACH

Table 1
Open-Ended Approach in terms of Stimulate Communication

The teacher…	M	SD	Verbal Interpretation
1. stimulates and monitors students' levels of engagement.	3.87	0.34	always
2. uses students' explanation for lesson content.	3.61	0.55	always
3. encourages and assists students to ask questions	3.68	0.53	always
4. gives students time to think before responding	3.76	0.49	always
5. intermittently initiates class discourse during the lesson	3.37	0.75	often
6. elicits many solutions or many methods from students	3.47	0.73	often
Mean	3.63	0.34	always

Legend: *3.50 – 4.00 - always, 2.50- 3.49 – often, 1.50 – 2.49 sometimes, 1.00 – 1.49 never*

The table presents the perception of students on how teachers use open-ended approaches in terms of stimulating communication in the classroom. The result of the study reveals that the teacher stimulates communication with a general mean of 3.63 and verbal interpretation of always. The standard deviation is relatively small (0.34), indicating little variation in students' perception of the teacher's communication-stimulating behavior.

The students perceived that the teacher always uses an open-ended approach effectively in terms of stimulating communication in the classroom. The teacher actively encourages students to engage with the topic with the mean of 3.87 and standard deviation of 0.34 with verbal interpretation of always. The teacher starts by presenting a real-life scenario, such as designing a floor plan for a house. The teacher asks questions to different students to share solutions and explain their work and provides feedback to each student while doing their tasks. When introducing a concept, the teacher asks students to describe what the students think, for example, "In two quadrilaterals that are similar and why the corresponding angles are congruent?". In this case, incorporating students' explanations into lesson content was established. While exploring quadrilaterals, the teachers always encourage students to ask questions. The teacher provides scaffolding techniques such as guiding questions, for example, "How can we determine if two quadrilaterals are similar? What criteria do we use to compare them?" "Are there any special relations or properties that exist within certain types of quadrilaterals? How can we use these properties to identify and classify them?". Before answering these questions, the teacher gives students ample time to think or process information to articulate their thoughts during discussion.

Moreover, an open-ended approach is a problem-solving approach that involves open-ended problems which provide multiple solutions and methods, so the teacher encourages students to generate multiple solutions or methods when working with the topics in a small group task, and this allows students to discuss and debate the task and to learn each other's perspectives. These strategies can enhance student engagement and learning in math class. The perception of students that the teacher always uses the open-ended approach, suggests that the student consistently believe their teacher effectively stimulates communication in the classroom.

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Table 2
Open-Ended Approach in terms of Support conceptual understanding

The teacher…	M	SD	Verbal Interpretation
1. encourages students to ask for help when needed	3.71	0.61	always
2. reminds students of previously learned problems with similar concepts	3.71	0.52	always
3. uses clarifying questions to guide students individually and in groups	3.71	0.57	always
4. asks a student to re-explain a peers' method.	3.24	0.63	often
5. consistently refers to the concepts being taught.	3.5	0.69	always
6. encourages students to generalize and analyze concepts	3.76	0.54	always
Mean	3.61	0.44	always

Legend: *3.50 – 4.00 - always, 2.50- 3.49 – often, 1.50 – 2.49 sometimes, 1.00 – 1.49 never*

The table reveals the perception of students on how teachers use open-ended approaches in terms of supporting conceptual understanding. Based on the results, the teacher is always supporting conceptual understanding with a general mean score of 3.61 (SD=0.44). The students perceived that the teacher encourages students to generalize and analyze concepts and has a mean score of 3.76 indicating highly effective practice with the standard deviation of 0.54. In the lesson exemplar, the teacher always encourages students to generalize what they have learned from the topic in the part of assimilation. To make sure that at the session the lesson objectives was learned by the students using this approach. While the teacher asks a student to re-explain a peer's method has a lowest mean of 3.24 (SD=0.63) indicating that this practice is often. The teacher sometimes used this strategy, the technique is called revoicing where teacher asked student to repeat other student's explanations, rephrase or elaborate the explanation and gave students chance to participate in the lesson. Although students need to be able to explain their thinking and learn from the thinking of others. Van de Walle et al, (2014) suggest that asking students to explain each other's thinking may not always be the most effective strategy. Instead, teachers should provide opportunities for students to engage in rich mathematical discussions and to analyze and critique various strategies and solutions.

Generally, the teacher is always using open-ended approach such as the teacher sometimes asked students with solution or done with the task to guide the other students, allowing those students ask for assistance, recapping previous lesson at the beginning of class and using open-questions to help them remember similar problems when they were stuck on the problem. The teacher used specific words like "so…"," then…", and "therefore…" to prompt students to provide more information or consider the concept in greater them. Those strategies are consistent in using an open-ended approach. However, there is still room for improvement, particularly in the area of promoting rich mathematical discussions and analysis of different strategies.

Table 3
Open-Ended Approach in terms of Stretch math thinking to the real world

The teacher…	M	SD	Verbal Interpretation
1. Lessons flows from Individual to group learning.	3.71	0.57	always
2. Uses a variety of examples	3.82	0.46	always
3. Asks students about the reason behind their strategy.	3.32	0.74	often
4. Encourages students to think about the text and context.	3.66	0.58	always
5. Develops students' reasoning skill	3.66	0.58	always
6. Apply solutions to students' everyday life	3.71	0.52	always
Mean	3.64	0.42	always

Legend: *3.50 – 4.00 - always, 2.50- 3.49 – often, 1.50 – 2.49 sometimes, 1.00 – 1.49 never*

Table 3 presents the perception of students to the teacher who is using an open-ended approach in terms of stretch math thinking to the real world. The teacher uses a variety of examples and has the highest mean score of

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3.82 indicating that it is always with standard deviation of 0.46. It is evident that the teacher is providing variety of examples to every lesson, asking students if there is other way or methods to solve the problem or giving them other technique to solve problem.

The teacher facilitating open-discussion to explore different approaches to a problem. Students were given hints which helped the other student to think about different ways the problem could be solved. The teacher asks students about the reason behind their strategy has a lowest mean of 3.32 (SD=0.74) indicating often or moderately effective practice. The teacher often asked questions how the students arrived to answer and helping them to examine the application of calculation to the real-life situations. Like for instance in proving Pythagorean Theorem, the student initially used method of using formula to prove theorem. However, the teacher asks the student to explain the real-world application of the theorem and how it is relevant in construction or engineering. This questioning prompt the student to revise the solution and find a more efficient and applicable way to prove the theorem that takes into account its real-world implications.

Overall, the mean score of 3.64 (SD=0.42) indicates that the teacher always using open-ended approach to stretch math thinking to the real world. For example, during the problem-solving exercises, the students were given time to think individually. When the teacher noticed that some students were struggling to solve the problem, then the teacher introduced collaborative learning, also the teacher asked students to read and analyze the problem or discussed them openly, then try to translate in Filipino or rephrase the problem so that they can solve the problem. The given problem in problem solving exercises is in the authentic format to relate or involve them in the problem. For example, "Using your bicycle, you travel a distance of 3 kilometers per hour. This morning, you rode to your bicycle and pedaled for 3 hours. How far did you travel this morning?". In this case, the students will think that they are in the situation. It became more meaningful for them to solve the problem because the students are involved in the problem.

Table 4
Open-Ended Approach in terms of Strengthen mathematical thinking

The teacher…	M	SD	Verbal Interpretation
1. Provide necessary information for students to solve the problem.	3.76	0.54	always
2. Challenge students with information one grade above their level.	3.45	0.76	Often
3. Questions flow from peripheral to deeper thinking.	3.53	0.6	always
4. Advocates for proof and accuracy	3.66	0.53	always
5. Encourage students to use math terms and symbols	3.68	0.62	always
6. Encourage students to use mathematical models	3.68	0.57	always
Mean	3.63	0.43	always

Legend: *3.50 – 4.00 - always, 2.50- 3.49 – often, 1.50 – 2.49 sometimes, 1.00 – 1.49 never*

This table shows the perception of the students in using an open-ended approach in terms of strengthening students' mathematical thinking. The overall mean score has 3.63 indicating students perceived that the teacher always implementing an open-ended approach. It is evident that the teacher increased the difficulty of problems, encouraged students to use previously established facts as reasons for their arguments and familiarized with the process of mathematical proof, encouraged students to use mathematical terms in the answer and corrected misuse when necessary, and helped students extract necessary information for problem-solving and use of models.

Providing necessary information for students to solve problems has the highest mean score with 3.76 which students perceive as always true. The teacher used questioning and hints to help students understand and solve problems, reminded the previously used methods and provided individual assistance during work. While the lowest mean score (3.45) is challenging students with information one grade above their level with the verbal interpretation of often true. Occasionally, students asked questions that required knowledge beyond the current grade level. The teacher responded by offering to explore the question together, but only if the class was interested. If the majority of the class decided not to pursue the questions, the teacher was still available to help individual students who wanted to tackle the problem on their own, either after or homework. Boaler (2014) stated that challenging students with more difficult problems can lead to improved mathematical thinking.

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The mean scores of other indicators are all above 3.5 with standard deviation of less than 1 which indicates that there is little variation on the students perception, indicating that students perceive the teacher to consistently use of the open-ended approach in terms of strengthening mathematical thinking. It is evident that the teacher starts to ask questions that tap into students' prior knowledge or personal experiences related to the topic. For example, "What are the examples of kite or where do you usually see a shape of trapezoid?", then teacher keep on asking questions that promote curiosity and investigation, then, guide students towards analyzing information they have gathered, the students need to reflect and connect to their own experiences. By structuring open-ended questions, it can be more comprehensive and have a meaningful understanding to the topic.

Table 5
Open-Ended Approach in terms of Successively Evaluate

The teacher…	M	SD	Verbal Interpretation
1. Maintain high expectations of ALL students	3.34	0.78	often
2. Write students' names beside their solution	3.08	0.88	often
3. Consistently observe the functional grasp of ideas	3.61	0.68	always
4. Encourage student to write their own summary	3.47	0.6	often
5. Provide time for student to be reflective and critical	3.71	0.65	always
6. Used the board to capture the entire lesson.	3.63	0.59	always
Mean	3.47	0.56	often

Legend: *3.50 – 4.00 - always, 2.50- 3.49 – often, 1.50 – 2.49 sometimes, 1.00 – 1.49 never*

The table above shows that the teacher is generally effective in using the open-ended approach in terms of evaluating student learning. The overall mean scores have 3.47 (SD=0.56) with the verbal interpretation of often which indicates that students perceived the teacher to be implementing this open-ended approach at least often.

The highest mean score is providing time for students to be reflective and critical with 3.71 and has standard deviation of 0.65 which perceive as always. It implies that the teacher is providing opportunities for students to think deeply about what they have learned and to evaluate their understanding, which can be effective to promote learning. For example, the teacher did this by facilitating a discussion after each group was presented. During the presentation, the class paid attention and followed along as the ideas were explained step-by-step, with a mathematical justification provided for each step. The presenter stayed at the board and welcomed questions and suggestions from their peers. In contrast, writing students' names beside their solution has the lowest mean score of 3.08 (SD=0.88) indicated as often. It means that this practice may not be seen as particularly by the students, but it can be useful for teachers to track student progress and provide feedback on individual work. According to Munroe (2015), writing names beside their solution has three purposes: to give students a sense of ownership and recognition, enhance their self-esteem and facilitate discussion by allowing other students to address their questions directly to the person who gave the method or suggestions.

Furthermore, it is evident also that the teacher often maintains high expectations for all students, emphasizing that they can achieve success and encouraging them to strive for excellence. The teacher always observes the functional grasp of ideas, monitoring students' understanding throughout the lesson. The teacher was able to identify areas where students may be struggling and provide additional support or clarification. The teacher always used the board to capture the entire lesson, visually representing the content covered. This approach helped students visualize the progression of ideas and concepts, aiding their understanding and retention.

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Table 6
Students' Proficiencies in terms of Make sense of problems and persevere in solving them

MP1	Pretest		Posttest		Level
	f	%	f	%	
8 to 10	16	42	33	87	Proficient
4 to 7	19	50	5	13	Developing
0 to 3	3	8		0	Emerging
Total	38	100	38	100	

The table shows the result of the pretest and post test levels for the mathematical practice of " Make sense of problems and persevere in solving them. Before the open-ended approach was implemented 42% were already proficient, while 50% were developing and 8% were emerging. After the implementation, the percentage who were proficient increased to 87%, while the percentage of students who were developing decreased to 13% because some problems were unable to provide a proper discussion/solution. No students was in the emerging level after the implemetation of open-ended approach. It implies that there is a significant improvement in students' ability to make sense of problems and persevere in solving them which involves analyzing a problem, developing a plan to solve it, and persevering through challenges to find a solution using an open-ended approach.

Figure 3
Answer of student A in problem 1 and 4 in making sense of problem and persevere in solving them pretest

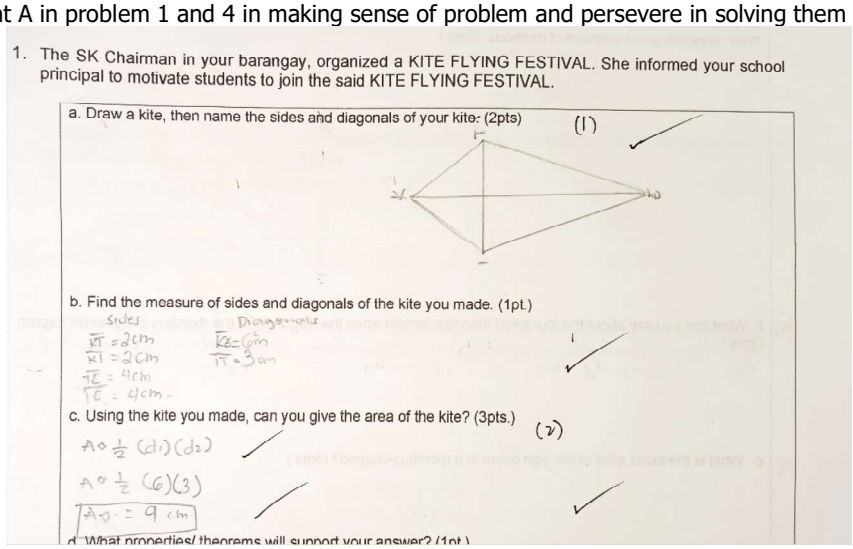

Figure 3 shows an example of student's answers at the developing. In letter c, the student was able to arrive at a solution without the need for background discussion. The student were able to give the formula for the area of kite and the corresponding value of the diagonal based from what she have drawn but do not have any discussion from the solution.

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Figure 4
Answer of student B in problem 1 and 4 in making sense of problem and persevere in solving them posttest

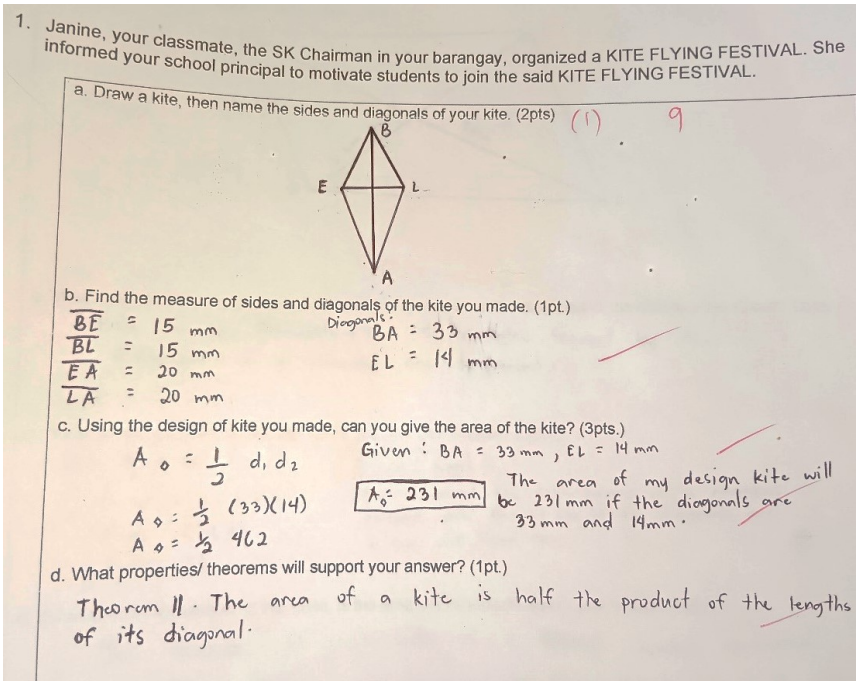

Figure 4 presents the students answer in making sense of problem and persevere in solving them at proficient level. The student's answer in finding the area of kite in letter c reveals that the student not only presents a solution but also includes discussion or evidences to support her answer. The student able to explain how to find the area of the kite she had made given the diagonals with 33mm and 14mm.

Futhermore, Number 1 is all about kite, they need to draw and measure the kite, then identify the area. The teacher provides enough time for students to read and analyze the problem, and identify the given information and what needs to be answered. The teacher prompt students to develop a plan by asking question. As the students work on their solutions, the teacher continously ask if their plans and solutions make sense and prompt them to explain their reasoning. Let the students answer the relationship of current problem or the related theorem might be used to solve the area of the kite.

Table 7
Students' Proficiencies in terms of reason abstractly and quantitatively

MP2	Pretest		Posttest		Level
	f	%	f	%	
8 to 10	32	84	37	97	Proficient
4 to 7	6	16	1	3	Developing
0 to 3		0		0	Emerging
Total	38	100	38	100	

The table shows the distribution of pretest and post-test scores for the mathematical practice of "Reason abstractly and quantitatively" among the students. Most of the students scored within the proficient level from 84%

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in the pretest to 97% in the post-test, while the other students score within the developing level from 16% to 3 % in the post.

Figure 5
Answer of student C in problem 2 in reason abstractly and quantitatively pretest

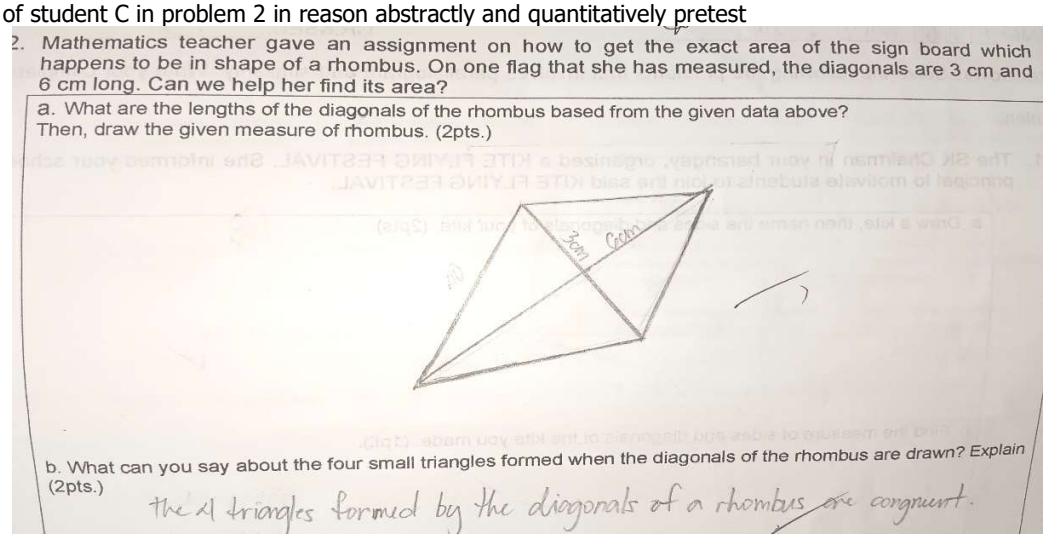
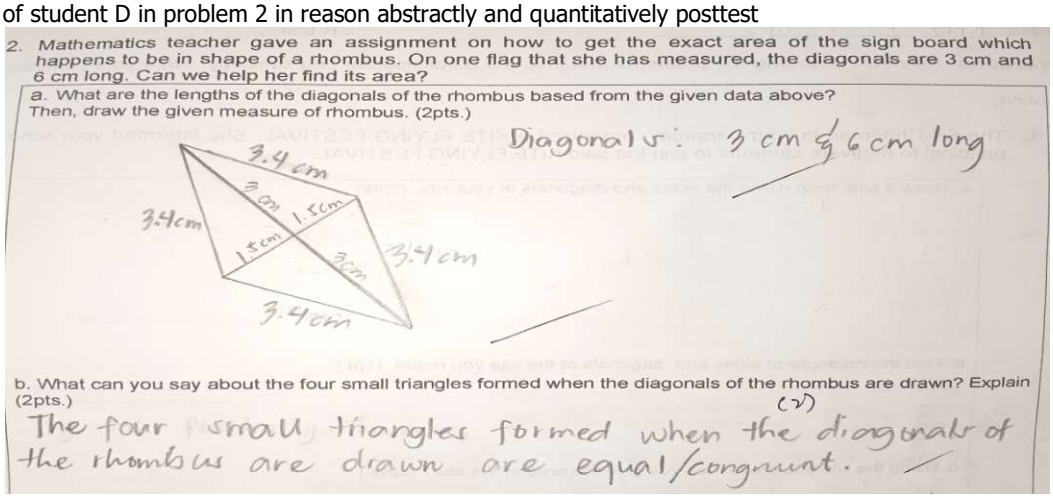


Figure 5 showed the student's answer in reason abstractly and quantitatively at developing level. The student was able to draw and identify the given measures but the illustration scaled incorrectly. The student was able to give simple explanation in the letter b question.

Figure 6
Answer of student D in problem 2 in reason abstractly and quantitatively posttest



The figure 6 reveals the answer of the student in reason abstractly and quantitatively at proficient level. The students' explanation about the four small triangles formed was clear and aligned with the given problem. The student also able to draw and identify the given measures correctly.

Moreover, it is evident that all questions related to reason abstractly and quantitatively include requires a student to attend to the meaning of quantities and relationships based from what they have drawn. The students can easily identify what being asked. The teacher asks student to explain the meaning of the symbols in the problem

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and in their solution. The teacher also asked questions if the student understand the relationship between symbols or quantities in the problem. It means that the students showed a strong ability to reason abstractly and quantitatively before and after using an open-ended approach.

Table 8
Students' Proficiencies in terms of Construct viable argument and critiquing the reasoning of others

MP3	Pretest		Posttest		Level
	f	%	f	%	
4 to 5	10	26	29	76	Proficient
2 to 3	18	47	9	24	Developing
0 to 1	10	26		0	Emerging
Total	38	100	38	100	

The table shows the pretest and posttest result for Mathematical Practice 3: Construct viable argument. The results indicated that there is an improvement in students' ability to construct viable arguments after using an open-ended approach. Before the implementation, 26% of the students were proficient, while after the implementation 76% of students reached the proficient level. It implies that the use of an open-ended approach was effective to develop students' proficiency in terms of constructing viable arguments. It is agreed by Hapiz and Rusdiana (2018) that the open-ended approach can improve students' argumentation skills in solving a mathematical problem because the open-ended approach provides opportunities for students to explore different solutions and explain their reasoning, which can lead to the development of argumentation skills.

Figure 7
Answer of student E in problem 4 in construct viable arguments and critiquing the reasoning of others pretest

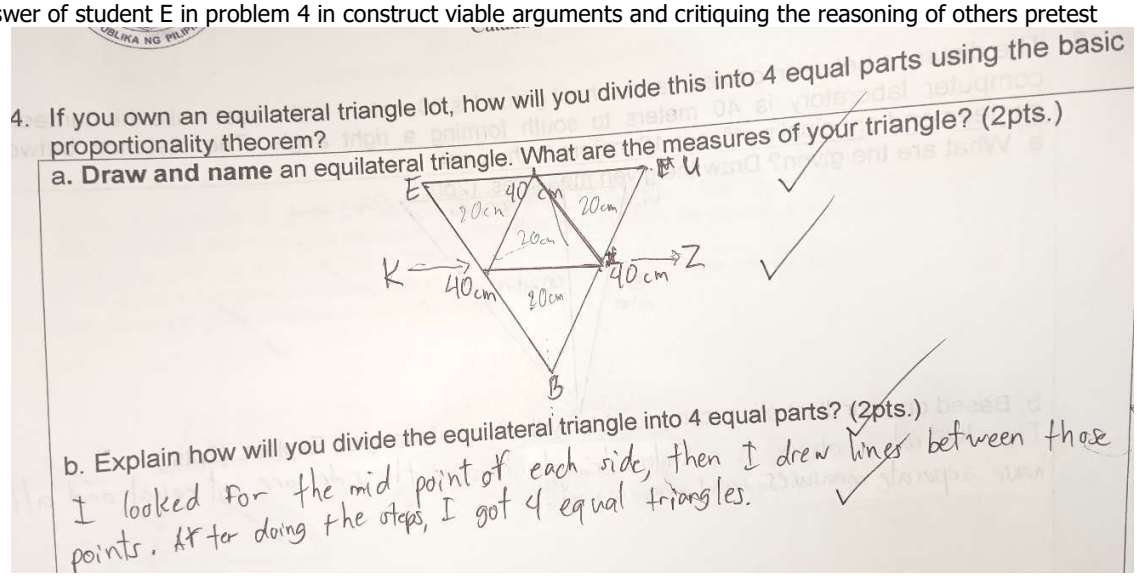

Figure 7 shows the student's answer to the mathematical practices no. 3 at developing level. The student provides a conjecture or solution but does not adequately explain or justify their reasoning behind it. In his answer in letter b, he provides a limited justification or partial reasoning. Prior on conducting experiment, the questions from number 3-5 in the instrument was not able to answer by the student, lack of time and knowledge to that particular problem. The students knew how to identify the relationship between the quantities but the students unable to justify their answer. The students can't give a clear and logical argument to their answer.

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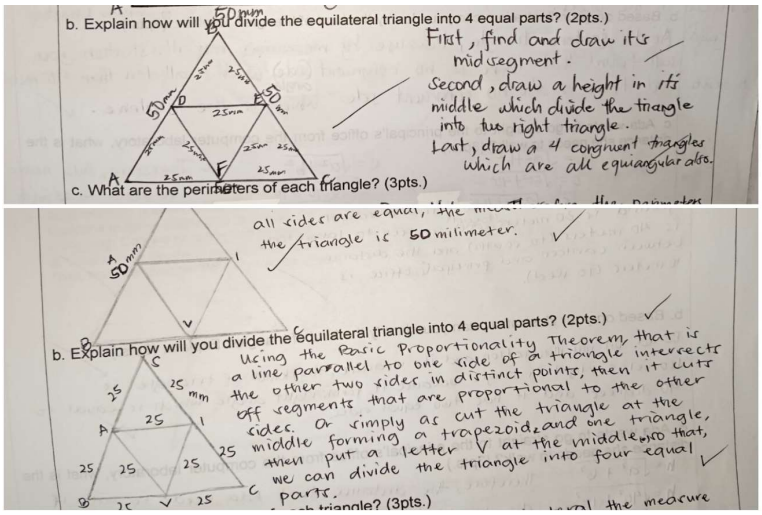
Figure 8
Answer of students F, G, and H in problem 4 in construct viable arguments and critiquing the reasoning of others posttest



The figure 8 shows the students' answer in constructing viable arguments and critiquing of others. The students demonstrate the ability to use conjectures and counterexamples effectively in their argumentation throughout the task. For instance, in student F, he was able to find and draw a midsegment first , through this he formed a two right angle lastly the student draw a 4 triangle that is equilateral but also equiangular. For student H "he cut the triangles into two equal parts using the median. Then, he cuts the other segments using the median".

After the implementation, no one in the class are in the emerging level and there was a decreased percentage on the developing. It implies that the teacher effectively used to asks the students reasoning, justify their answers during class discussion and elaborate and rephrase the reasoning or explanations of others to have a significant improvement in constructing viable arguments and critiquing the reasoning of others.

Table 9
Students' Proficiencies in terms of model with mathematics

MP4	Pretest		Posttest		Level
	f	%	f	%	
4 to 5	25	66	37	97	Proficient
2 to 3	13	34	1	3	Developing
0 to 1		0		0	Emerging
Total	38	100	38	100	

Based on the table, there is a significant development in the level of proficiency in modeling in mathematics from the pretest and post-test. In the pretest, 66% of the students were at the proficient level, while 34% were at the developing level. The students are able to model the problem such as draw a kite, draw an equilateral triangle, provide a measure and identifying the given. This show that student creating a model that the student will see the relationship between the quantities and they can represent the problem and solution.

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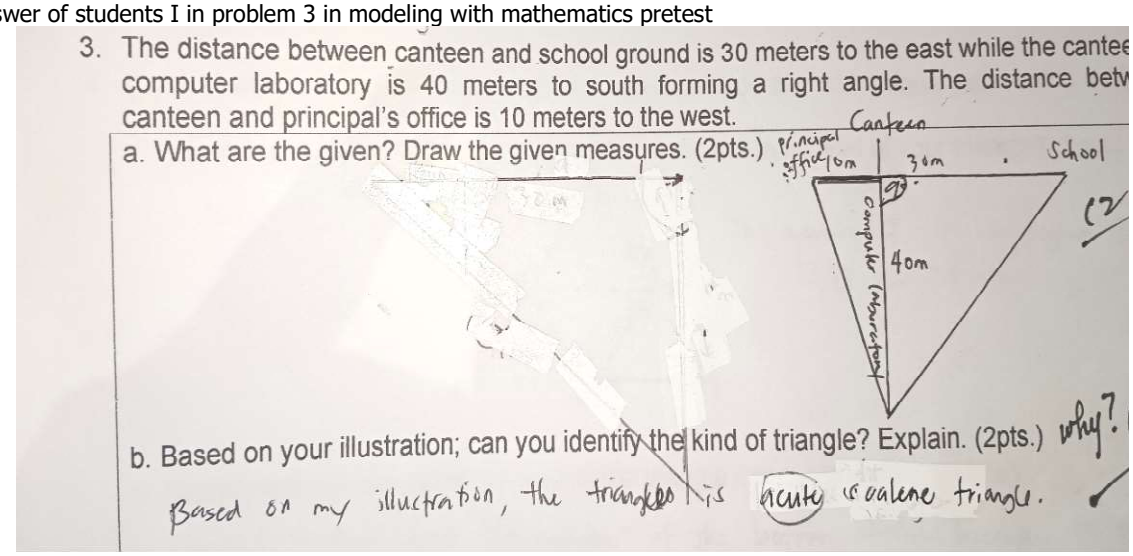
Figure 9
Answer of students I in problem 3 in modeling with mathematics pretest



The figure 9 shows how the students creates a model but fails to explain how it relates to the scenario or how it can be used to solve the problem. It was interpreted as developing level because the student creates a model but it does not effectively enhance the clarity of the given scenario.

Figure 10
Answer of students J in problem 3 in modeling with mathematics posttest

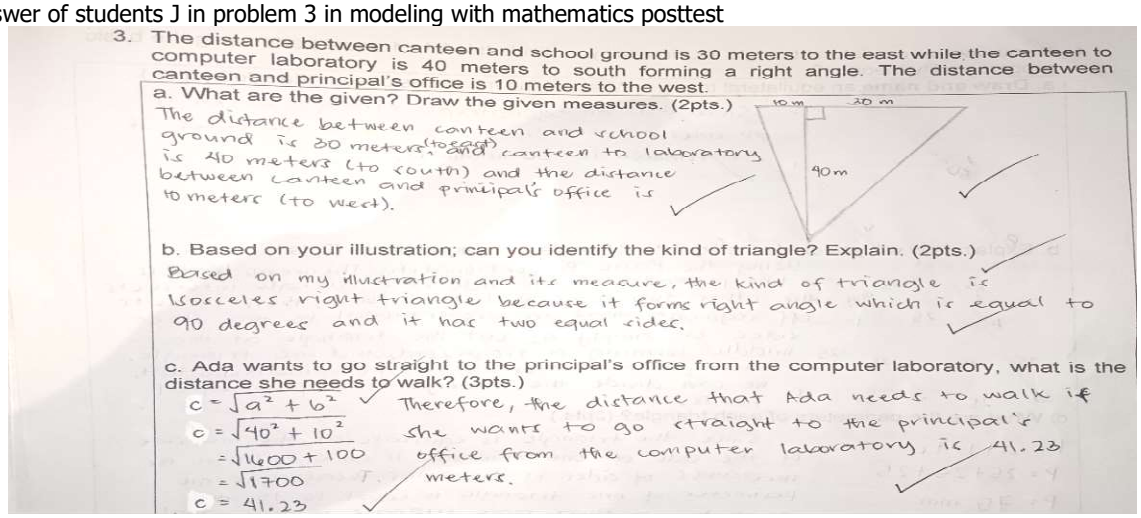


Figure 10 shows the student's answer in problem 3 in modeling with mathematics and it was interpreted as proficient. It is evident that the student creates a mathematical model that clearly represents the scenario, simplifies the problem and can be used to different solution. The student demonstrates its usefulness in finding the solution to the distance of two locations.

Moreover, in the posttest, most of the students reached the proficient and only one student remained at the developing level. This improvement implies that the use of an open-ended approach can develop students'

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performance in modeling mathematics by demonstration and provides students experiences with the use of mathematical model. The teacher allowed them to draw or illustrate the real-life problem before arriving to solution. For instance, in the problem of triangle similarity such as dividing an equilateral triangle into four parts, the students will figure out that these four parts are also equal because the triangle is equilateral which are equal in side.

Table 10
Students' Proficiencies in terms of model with mathematics

MP5	Pretest		Posttest		Level
	f	%	f	%	
4 to 5	12	32	34	89	Proficient
2 to 3	19	50	4	11	Developing
0 to 1	7	18		0	Emerging
Total	38	100	38	100	

The table above shows the significant improvement in the posttest scores for the use of appropriate tools strategically with 89% of the students reaching the proficient level. Before the implementation, only 32% was able to use appropriately the protractor to measure an angle and also ruler, the students do not know the proper use of inches and centimeters or millimeters. It has been found out that student estimate their illustration or used some concrete objects to draw the given.

Figure 11
Answer of students J in problem 2 in using appropriate tools strategically pretest

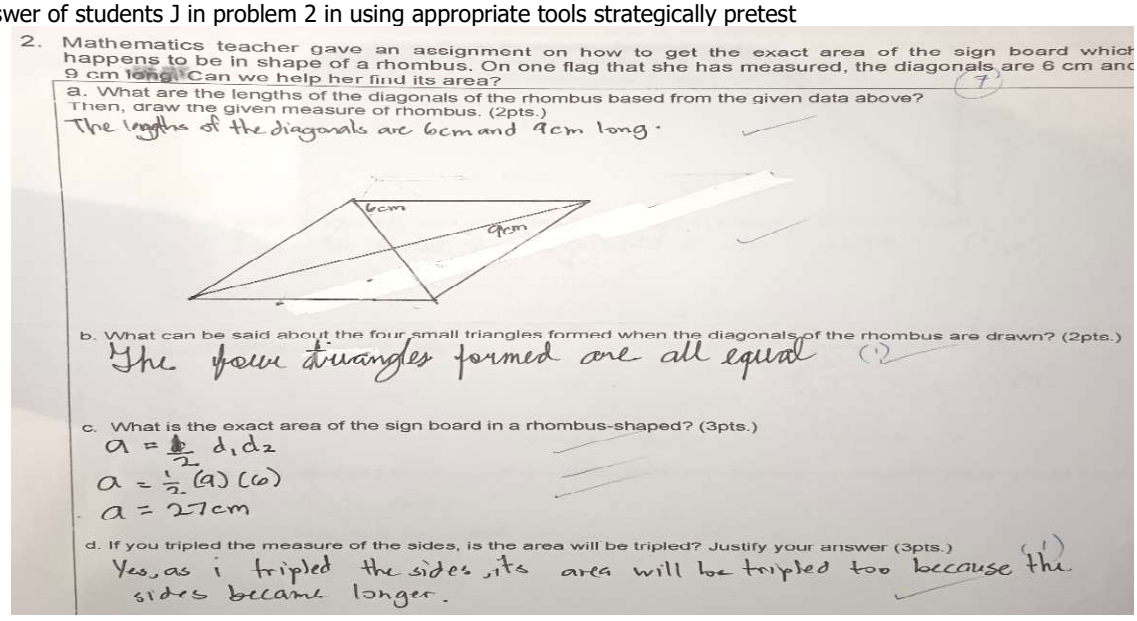

Figure 11 shows how student draw the figure without the use of appropriate tools, he only estimated the measure of the figure. The shape is not a rhombus however it looks like a kite but then, the student explains that four triangles formed are all equal because of the figure drawn. It was interpreted in the developing level.

During the experiment, the teacher demonstrates and provide students experiences with the various math tools that helped them to solve problems. Teacher asks the students why they chose the tools to solve the problems.

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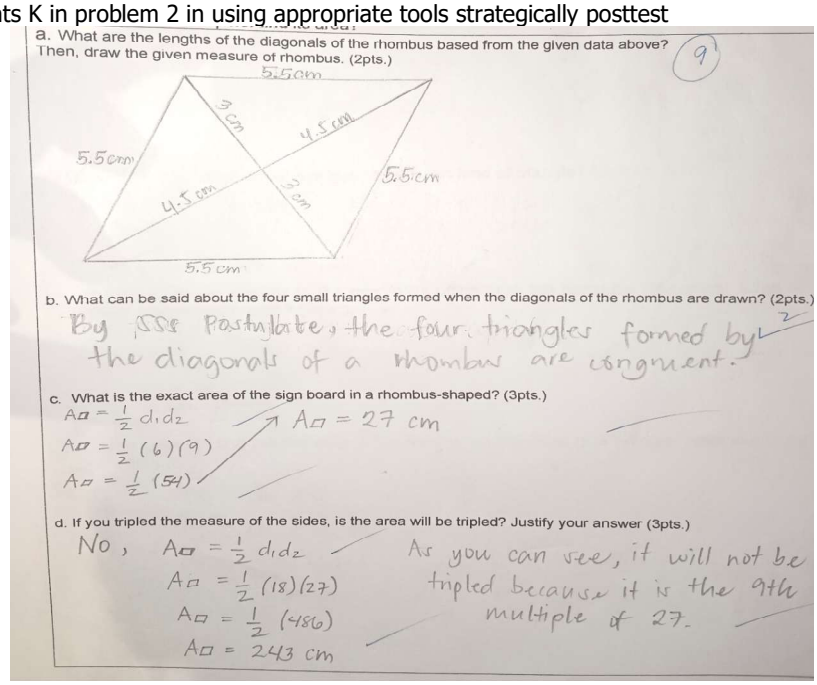
Figure 12
Answer of students K in problem 2 in using appropriate tools strategically posttest



The figure 12 presents the student's answer in using appropriate tools strategically at proficient level. It is evident that student used an appropriate tool such as rulers to illustrate the given measure and formed a correct rhombus that are equal in sides. Then, connect to the problem to arrive at a solution.

Therefore, it indicates that the students have learned how to choose and use appropriate tools to solve problems effectively and efficiently. According to NCTM (2014), using appropriate tools strategically is an essential component of mathematical proficiency.

Table 11
Students' Proficiencies in terms of attend to precision

MP6	Pretest		Posttest		Level
	f	%	f	%	
7 to 9	7	18	31	82	Proficient
4 to 6	14	37	7	18	Developing
0 to 3	17	45		0	Emerging
Total	38	100	38	100	

The table shows the pretest and post test scores for attending to precision with corresponding levels of proficiency. It shows that there is an increase in the number of students who reached the proficient level after the implementation of the open-ended approach. In the pretest, 18% of the students reached the proficient level but, in the posttest, it increased to 82%. The number of students who are developing also decreased from 37% in the pretest to 18% in the posttest.

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Figure 13
Answer of student L in problem 1 in attend to precision pretest

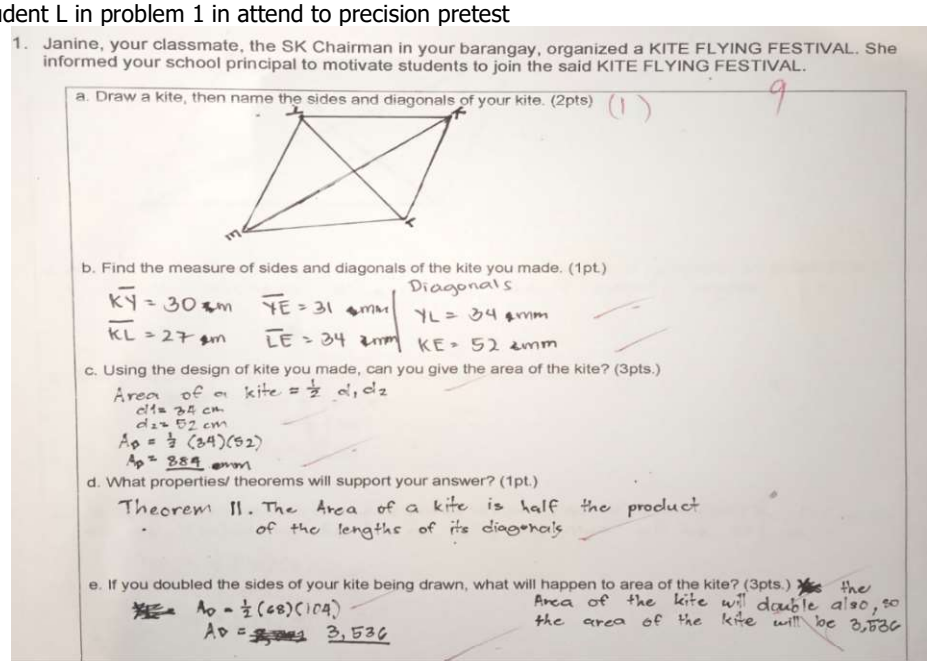

Figure 13 is an example of student answer that show developing level. It was evident that the student initially uses symbols, label, or units but fails to maintain consistency or accuracy throughout their solution.

Figure 14
Answer of student L in problem 1 in attend to precision posttest

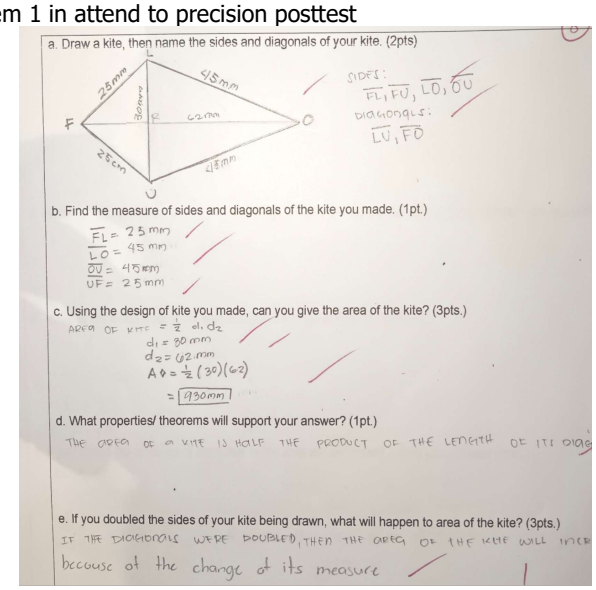

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Figure 14 is an example of student answer that shows proficient level. It was evident that the student used appropriate label or units in the solution. The student appropriately and effectively used a math symbols and terminology like symbol for line segment, diagonal and millimeters, to express their mathematical ideas and operations.

Furthermore, there are problems identified that the student doesn't use precise vocabulary when communicating ideas. Most of the answers were not properly labeled before the implementation. During the implementation, the teacher consistently uses and models correct content terminology such as the units, symbols and quantities. Thus, the results revealed there was a significant improvement in the students' proficiencies after using the open-ended approach. The improvement in performance highlights the importance of explicitly teaching precision in mathematics as it is crucial skill for understanding mathematical concepts and solving problems accurately.

Table 12

Students' Proficiencies in terms of ability to look for and make use of structure

MP7	Pretest		Posttest		Level
	f	%	f	%	
5 to 6	4	11	35	92	Proficient
3 to 4	7	18	3	8	Developing
0 to 2	27	71		0	Emerging
Total	38	100	38	100	

The table shows the pretest and posttest results of using an open-ended approach to develop students' proficiencies in terms of the ability to look for and make use of structure. The pretest results were 11% of the student scored in the proficiency level while 71% were in the emerging level, indicating a need for improvement in this mathematical practice. It has been found out the student require to look back on the previously learned concepts in mathematics. Like for instance, in finding the perimeters of triangle, the area of rhombus which also can be solved by the area of kite or the distance between two points or two locations. From this results in the pretest, the teacher encourages students to look for something they recognize and have students apply the information in identifying solution paths. Providing opportunities for students to explore multiple representations of mathematical ideas and to make connections between them. Encouraging students to look for similarities and differences between problems and to use their understanding of these patterns to make sense problems. However, in the post test, 92% of the students scored at the proficient level, there was a significant improvement from the pretest. The improvement in students' ability to look for and make use of structure shown in the post test can be attributed to the use of an open-ended approach that emphasizes visualization and exploratory methods.

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Figure 15
Answer of student M in problem 5 in look for and make use of structure pretest

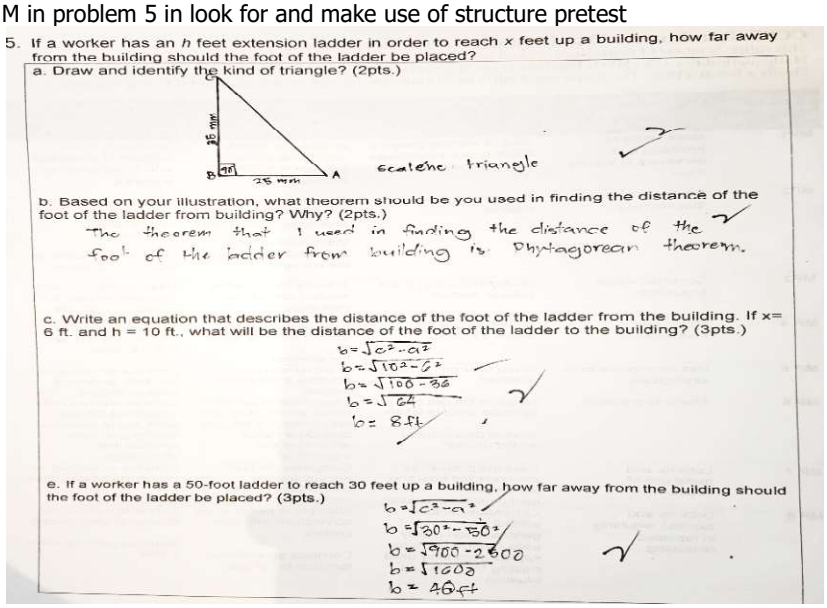

Figure 15 shows the student's answer in developing level. It was evident that the student completes the task but fails to draw to connections to other mathematical content or structures they have learned. The student was able to give a complete solution in the problem 5 for letter c and d but not able to connect to prior structure and the mathematical structure was not fully integrated to the solution.

Figure 16
Answer of student M in problem 5 in look for and make use of structure posttest

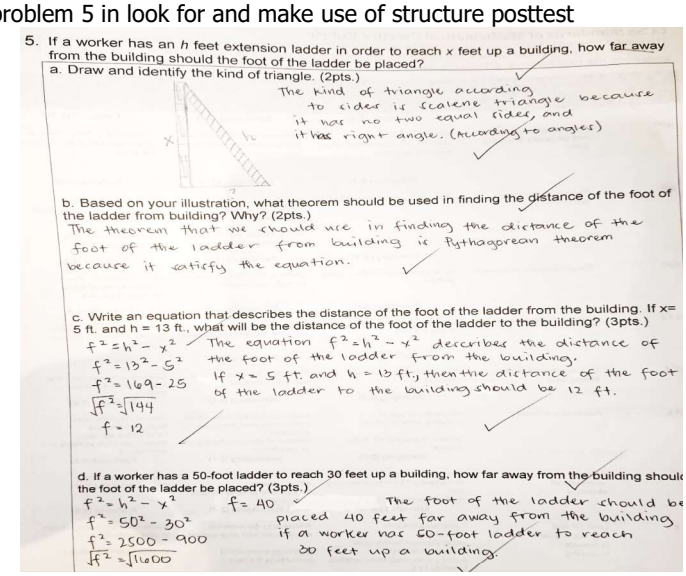

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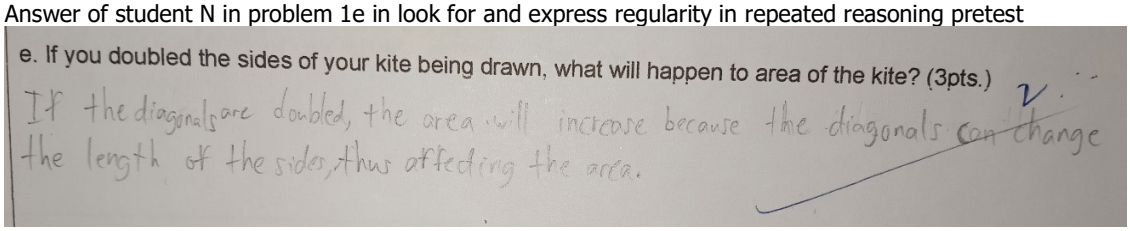
Figure 16 is an example of student answer at proficient level. It was evident that the student answer describes the connections of solutions to the mathematical structure and explains the use of distance or the use of Pythagorean theorem to solve the problem.

Table 13
Students' Proficiencies in terms of Look for and express regularity in repeated reasoning

MP7	Pretest		Posttest		Level
	f	%	f	%	
4 to 5	8	21	32	84	Proficient
2 to 3	26	68	6	16	Developing
0 to 1	4	11		0	Emerging
Total	38	100	38	100	

The table presented shows the pretest and post test results of the students in eighth mathematical practices, which is to "look for and express regularity in repeated reasoning.". 81% of the students achieved proficiency level on the posttest compared to 21% on the pretest. Additionally, the percentage of students in the developing level decreased from 68% to 16% after using the open-ended approach.

Figure 17
Answer of student N in problem 1e in look for and express regularity in repeated reasoning pretest



The figure 17 presents the student answer in look for and express regularity in repeated reasoning at developing level. In this case, the student has a limited explanation to the given problem but fails to include the solution to prove that if the diagonals are doubled, the area will also be doubled. The student reasoned-out that the area of kite change its measure increasingly.

Figure 18
Answer of student O in problem 1 in look for and express regularity in repeated reasoning posttest

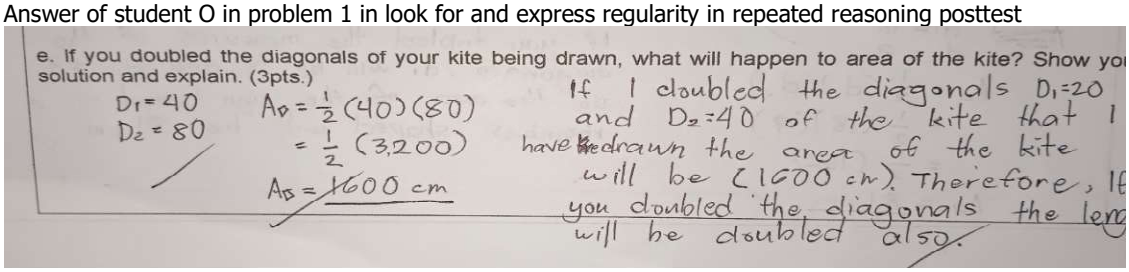


Figure 18 is an example of student answer that shows proficient. It was evident that the student wrote a solution and was able to generalize the given problem. The student explains if the diagonals of kite was doubled, the area of the kite also doubled. It indicates that in the next given problem, the student no need to provide a solution to generalize the pattern.

These results imply that the open-ended approach successfully supported student learning to look for and express regularity in repeated reasoning. Before the experiment, the 79% were below proficient level, most of the students cannot see patterns or relationships on the given problem therefore the student cannot construct the

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generalization. The students' do not write anything for the last part of the problem. There is no therefore statement to answer the what is being asked in the problem. By using open-ended approach, the teachers reminded students to generalize the problem, asks the students what math relationships or patterns can be used to assist in making sense of the problem, asks the predictions about the solutions at midpoints throughout the solution process and assists them in creating generalizations based on repetition in thinking and procedures.

The improvement in performance highlights the importance of explicitly teaching students to identify patterns and regularities in mathematics.

Table 14

Difference between Pretest and Posttest scores using Open-Ended Approach.

	Pretest		Posttest		t	df	Sig. (2-tailed)
	M	SD	M	SD			
Make sense of problems and persevere in solving them	6.74	1.95	8.82	1.01	-7.475	37	0.000
Reason abstractly and quantitatively	4.29	0.87	4.63	0.54	-2.122	37	0.041
Construct viable argument	2.63	1.32	4.18	0.80	-7.967	37	0.000
Model with mathematics	3.92	0.85	4.37	0.54	-3.468	37	0.001
Use appropriate tools strategically	2.66	1.21	4.26	0.64	-8.422	37	0.000
Attend to precision	4.29	2.28	7.45	1.13	-10.589	37	0.000
Look for and make use of structure	2.18	1.49	4.66	0.91	-11.642	37	0.000
Look for and express regularity in repeated reasoning	2.82	1.20	4.32	0.74	-8.528	37	0.000

Based on the given table there was a significant difference between the pre-test and post-test scores performance in the students' proficiencies who were exposed to an open-ended approach based on stimulating communication, supporting conceptual understanding, stretching math thinking to the real world, strengthening mathematical thinking, and successively evaluating. This means that the students were able to develop their proficiencies in geometry and that the use of an open-ended approach is effective in developing their proficiencies. Most of the proficiencies were marked as highly significant at 0.0000 but the other proficiencies were marked as significant at 0.041 and 0.001.

Most of the proficiencies' pre-test and post-test scores of those who were exposed to an open-ended approach increase its level of performance from developing to proficient, retained to a proficient level of performance in terms of reason abstractly and quantitatively and model with mathematics, while the table shows that in looking for and making use of structure, using an open-ended approach increases its level of performance from emerging to proficient.

Specifically, the largest improvements were observed in Make sense of problems and persevere in solving them, use appropriate tools strategically and attend to precision. These results suggests that the open-ended approach used in the learning process was effective in developing these mathematical proficiencies. The open-ended approach provides students with opportunities to explore, experiment, and construct their own understanding of mathematical concepts rather than simply memorizing rules or algorithms.

For example, In the Make sense of problems and persevere in solving them, the open-ended approach encourages to actively engage with challenging problems, apply multiple of strategies, and persist through difficulties. This approach helps students to develop problem-solving skills that are transferable to various situations.

In use appropriate tools strategically, the open-ended approach allows students to choose and use various tools and technology to solve problems efficiently and effectively.

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In attend to precision, the open-ended approach emphasizes the importance of precise communication and reasoning in mathematics. For example, students are encouraged to explain their thinking clearly, use appropriate mathematical notation and check their work for accuracy.

In addition, the open-ended approach promotes active and engaging learning environment. The teacher starts a lesson always present a real-world problem that do not have a single correct answer. These problems required students to analyze and interpret information, make decisions and apply mathematical concepts and explore different approaches while solving problem. The open-ended approach stimulates critical thinking skills and reasoning skills. Students must analyze problems, identify patterns and relationships, make connections and draw logical conclusion. The teacher always required the students to justify their reasoning and provide evidence to support their solutions. The open-ended approach always involved collaborative work and encouraged students to communicate their ideas, strategies and reasoning with their peers. In this case, the student explores unconventional approaches to the given problem, think outside the box, and come up with unique solutions. Therefore, the open-ended values the process of problem-solving and critical thinking rather than focusing solely on the final answer or solution. Emphasize the process over product.

Overall, the open-ended approach in Geometry empowers students to take ownership of their learning, develop higher-order thinking skills, and cultivate a deeper understanding of mathematical concepts through exploration, problem-solving, collaboration and reflection. It was evident that open-ended approach is effective in developing students' proficiencies in Geometry. The result of this research is related to English and Sriraman (2015) who found that students who were taught using an open-ended approach to mathematics showed improvements in their problem-solving abilities compared to those who were taught using a traditional approach.

Summary, Conclusions, and Recommendations

This study was designed to determine whether the use of open-ended approach may develop students' proficiencies in Geometry. The study yielded the following findings.The teacher is generally effective in implementing an open-ended approach in the classroom. However, there is a need for improvement, particularly in promoting rich mathematical discussions, analyzing different strategies, challenging students to more difficult problems, and writing students' names beside their solutions. The study's results highlight the effectiveness of using open-ended approach in teaching geometry. Therefore, eduactors should consider promoting the use of open-ended tasks and activities in mathematics curriculum. This approach can stimulate communication, support conceptual understanding and strengthen mathematical thinking, leading to improved student performance.

The pretest and posttest scores performance in Geometry of students who were exposed to an open-ended approach have developed their proficiencies. There was a significant improvement in student's performance in all of the proficiencies. The increase in the level of performance varied from develop ping to proficient, retained to a proficient level of performance, and emerging to proficient. Teachers should integrate these practices throughout the mathematics curriculum to cultivate well-rounded mathematical proficiency in students.

There were highly significant improvements in students' mathematical proficiency in various mathematical practices after using the open-ended approach. The use of an open-ended approach is effective in improving students' abilities to make sense of problems and persevere in solving them, reason abstractly and quantitatively, construct viable arguments, a model with mathematics, use appropriate tools strategically, attend to precision, look for and make use of structure and look for and express regularity in repeated reasoning.

Based on the conclusion, the following recommendations are given. First, Mathematics teachers should implement more open-ended approach in teaching mathematics and use this approach frequently in their strategies to help students develop their problem-solving skills and mathematical practices. Mathematics teachers may provide more challenging problems or encourage students to explore more complex mathematical concepts. Second, Carefully choice topics that can be thought using open-ended approach not only to the field of Geometry but also to other branch of Mathematics.Moreover, While the current findings suggest that open-ended approach was effective in developing students' proficiencies, it would be useful for the future researcher to conduct more longitudinal studies to explore the long-term impact of open-ended approach on the outcomes

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